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DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

LARGE-AMPLITUDE MULTIMODE RESPONSE OF CLAMPED RECTANGULAR PANELS TO ACOUSTIC EXCITATION

By

Chuh Mei, Principal Investigator

Interim Technical Report
For the period October 1, 1980 - September 30, 1981

Prepared for the
Air Force Flight Dynamics Laboratory
Wright-Patterson Air Force Base
Ohio

Under
Grant No. AFOSR-80-0107
Howard F. Wolfe, Program Manager
AFWAL/FIBED
Acoustics and Sonic Fatigue Group

February 1982

OLD DOMINION UNIVERSITY RESEARCH FOUNDATION

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A mathematical formulation and the solution procedure for rectangular panels under broadband random acoustic excitation are presented. Large-amplitude effect and multiple modes are included in the formulation. The generalized mass matrix and the generalized linear stiffness matrix using 15 terms in the assumed panel deflection function are derived. Subroutine programs that generate the mass and linear stiffness matrices have been developed. Continuing research efforts are outlined.

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FOREWORD

This report contains the research effort on large-amplitude multimode response of clamped rectangular panels to acoustic excitation during the period from October 1, 1980 to September 30, 1981. The work was performed at the Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia. The research was sponsored by the Air Force Office of Scientific Research (AFSC), Department of the Air Force, under Grant AFOSR-80-0107. The work was monitored under the supervision of Howard F. Wolfe, Technical Manager, Acoustics and Sonic Fatique Group, Air Force Wright Aeronautical Laboratories, and Dr. Alan H. Rosenstein, AFOSR/NE, Program Manager, Directorate of Aerospace Sciences, AFSC. The author gratefully acknowledges the encouragement and assistance from Mr. Howard F. Wolfe and Dr. Donald B. Paul of AFWAL.

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By

Chuh Mei

INTRODUCTION

Acoustically induced fatigue failures in structural components have resulted in unacceptable maintenance and inspection burdens associated with aircraft and missile operation. In some cases, sonic fatigue failures have resulted in major structural redesigns and aircraft modifications. Thus, accurate prediction methods are needed to determine the fatigue life of structures.

Many analytical and experimental programs to develop sonic fatigue design criteria, however, have repeatedly shown a poor comparison between measured and calculated maximum RMS stress/strain (refs. 1, 2). Deviations in excess of 100 percent are not uncommon. Large deflection nonlinearity has been identified as a major factor for the enormous discrepancy between test data and computed results (ref. 3). A test program was conducted recently to check the analytical effort for the large-amplitude, single-mode response reported in reference 3. The acoustic response tests were performed in the Wideband Acoustic Facility at Wright-Patterson Air Force Base. A comparison of the results from two panels is shown in figure 1. The prediction of random responses is much improved with the single-mode computational method, especially at high excitation levels. Test results (fig. 2) also showed that there are more than one mode responding. Multiple modes were also observed by White in experimental studies on aluminum and carbon fiber-reinforced plastics (CFRP) plates under acoustic loadings (ref. 4). White also showed that the fundamental mode responded significantly and contributed more than 80 percent of the total mean-square strain response; higher modes, up to third or fourth modes, account for 95% or more of the total mean-square strain response. In order to have an accurate prediction of the random response of a structure, multiple modes should be used in the formulation.

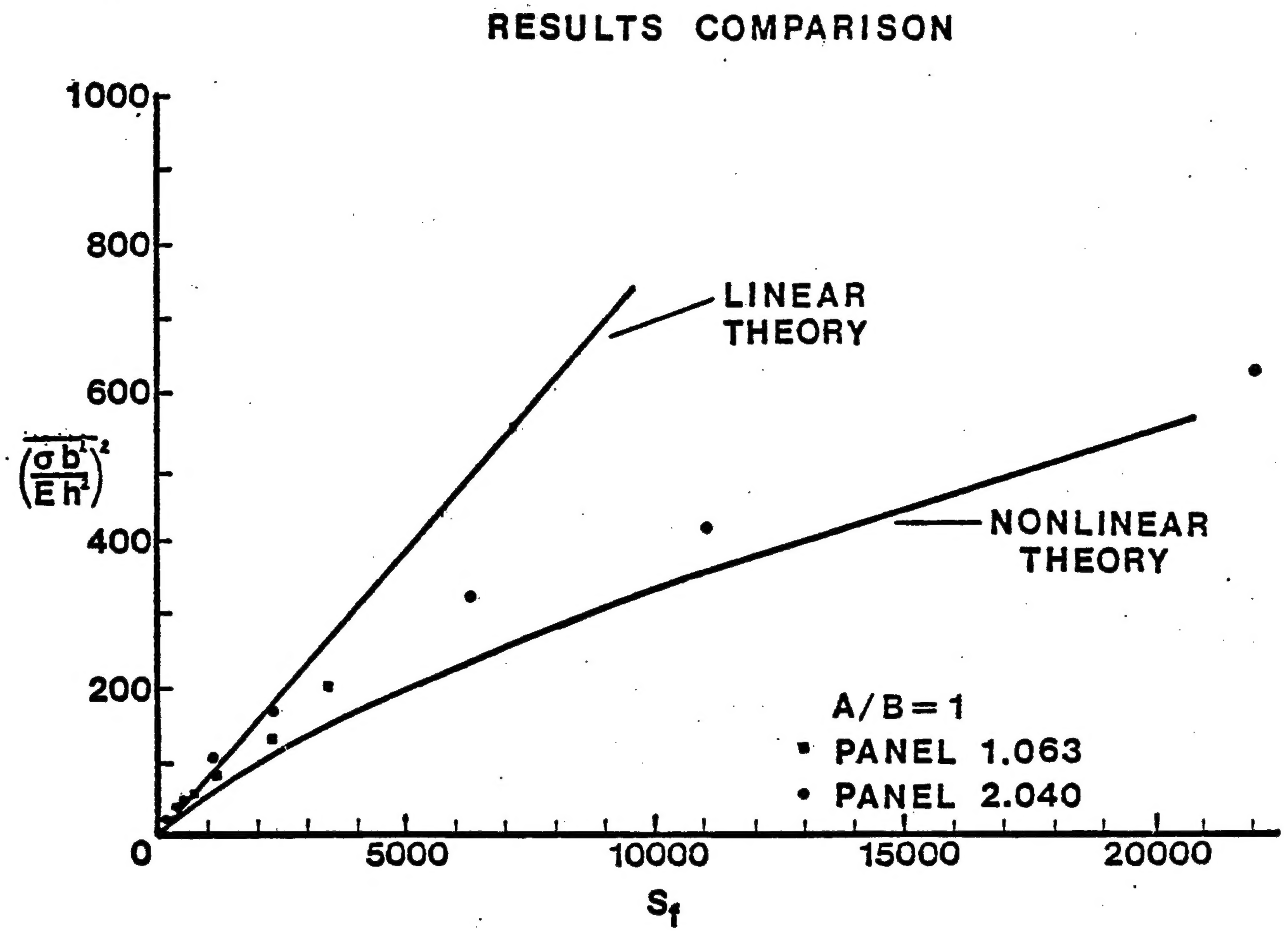

Figure 1. Comparison of analytical and experimental mean-square stresses of clamped, square, aluminum panels.

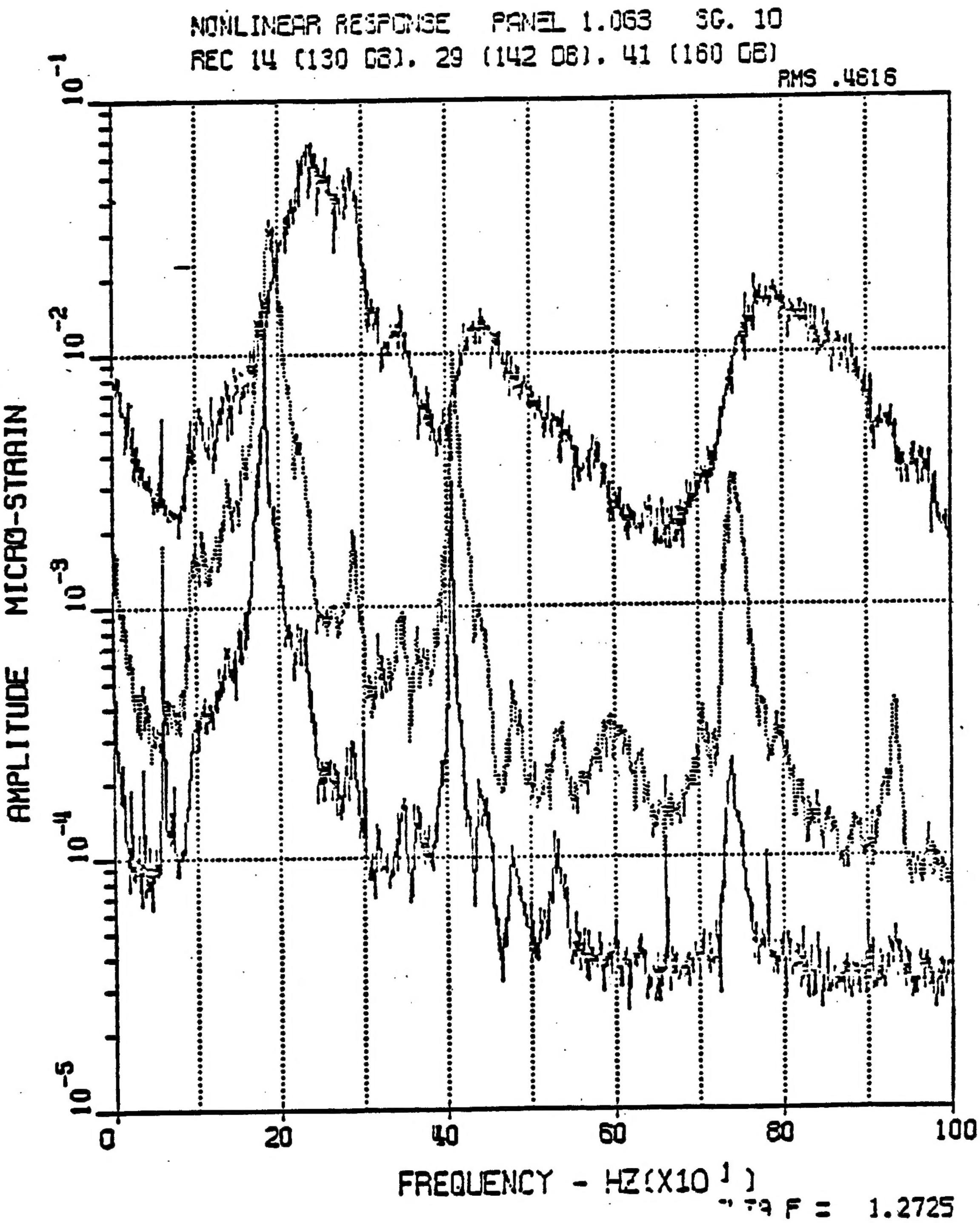

Figure 2. Strain response at three different SPL's.

NOMENCLATURE

a,b	panel length and width
C	generalized damping
D	flexural rigidity
E	Young's modulus
$f_m(x), g_n(y)$	displacement functions, eq. (21)
F	airy stress function
h	panel thickness
K	generalized stiffness
L	mathematical operator, eq. (1)
M	generalized mass
p	pressure
q	normal ccoordinate
r	length-to-width ratio, a/b
$S_p(\omega)$	cross-spectral density of p(t)
t	time
w	lateral deflection
W	generalized displacement
x, y	coordinates
β	vector function, eq. (16)
ζ	damping ratio, c/c_o
ϕ	normal mode
ω	linear frequency
Ω	equivalent linear or nonlinear frequency

<u>Subscripts</u>

EL	equivalent linear
L	linear

MATHEMATICAL FORMULATION AND SOLUTION PROCEDURE

The governing equations of a rectangular, isotropic plate undergoing large-deflection motions, neglecting the effects of both inplane and rotatory inertia forces, are (refs. 5, 6)

$$L(w,F) = D\nabla^4 w + \rho h w_{,tt} + g w_{,t}$$

$$- h(F_{,yy} w_{,xx} + F_{,xx} w_{,yy} - 2 F_{,xy} w_{,xy})$$

$$- p(t) = 0 \tag{1}$$

$$\nabla^4 F = E(w^2_{,xy} - w_{,xx} w_{,yy}) \tag{2}$$

where a comma denotes the partial differentiation with respect to the corresponding variable, w is the lateral deflection, F is the stress function, D is the flexural rigidity, ρ is the mass density, h is the plate thickness, p is the pressure, E is the Young's modulus, and g is the viscous damping.

The lateral deflection is assumed as

$$w(x,y,t) = h \sum_m \sum_n W_{mn}(t)\, f_m(x)\, g_n(y) \qquad m,n = 1,2,3,\ldots \tag{3}$$

where the functions $f_m(x)$ and $g_n(y)$ are so chosen that they satisfy the boundary conditions. By solving the compatibility equation, equation (2), the stress function can then be determined as

$$F = \overline{N}_x \frac{y^2}{2} + \overline{N}_y \frac{x^2}{2} + Eh^2 \sum_i \sum_j F_{ij}\, N_i(x)\, M_j(y)$$

$$i,j = 0,1,2,\ldots \tag{4}$$

A quasi-exact solution has been obtained by Paul for thermal postbuckling of a clamped, rectangular plate. The expressions for the coefficients $\overline{N}_x$, $\overline{N}_y$, and F_{ij} can be found in reference 7.

Apply the Bubnov-Galerkin method to the equation of motion in deflection, equation (1), as

$$\iint L(w,F)\, f_r\, g_s\, dxdy = 0 \qquad r,s = 1,2,3... \tag{5}$$

After performing the integration over the total area of the panel, a set of nonlinear, time-differential equations is obtained and can be written in matrix form as

$$[M]\{\ddot{W}\} + [C]\{\dot{W}\} + [K]_L\{W\} + \{\beta(W)\} = \{p(t)\} \tag{6}$$

where the matrices $[M]$, $[C]$, and $[K]_L$ are the generalized mass, damping, and linear stiffness matrices, respectively, and $\{\beta(W)\}$ is a vector function, cubic in the generalized displacements $\{W\}$.

An equivalent linear set of equations to equation (6) may be defined as (refs. 8-13):

$$[M]\{\ddot{W}\} + [C]\{\dot{W}\} + \left([K]_L + [K]_{EL}\right)\{W\} = \{p(t)\} \tag{7a}$$

or

$$[M]\{\ddot{W}\} + [C]\{\dot{W}\} + [K]\{W\} = \{p(t)\} \tag{7b}$$

where the elements of the equivalent linear stiffness matrix $[K]_{EL}$ can be obtained from the expression

$$\left(K_{EL}\right)_{ij} = E\left[\frac{\partial\beta_j(W)}{\partial W_i}\right] \qquad i,j = 1,2,3,... \tag{8}$$

where $E[\ ]$ is an expected value operator.

To determine the mean-square generalized displacements $\overline{W_j^2}$ in equation (7), an iterative solution procedure is introduced. The undamped linear equation of equation (7a) is solved first. This requires the determination of the eigenvalues and eigenvectors of the undamped linear equation

$$\omega_j^2 \, [M] \, \{\phi\}_j = [K]_L \, \{\phi\}_j \tag{9}$$

where ω_j is the frequency of vibration and $\{\phi\}_j$ is the corresponding normal mode shape based on linear theory.

Apply a coordinate transformation, from the generalized displacements to the normal coordinates, by

$$\underset{m\times 1}{\{W\}} = \underset{m\times n}{[\phi]} \; \underset{n\times 1}{\{q\}} \qquad n < m \tag{10}$$

in which each column of $[\phi]$ is a modal column of the linear system, and $\{q\}$ represents the normal coordinates. Substituting equation (10) into the damped linear equation of equation (7a) and premultiplying by the transpose of $[\phi]$, it becomes

$$\lceil M \rfloor \{\ddot{q}\} + \lceil C \rfloor \{\dot{q}\} + \lceil K \rfloor_L \{q\} = \{P(t)\} \tag{11}$$

where

$$\begin{aligned}
\lceil M \rfloor &= [\phi]^T \, [M] \, [\phi] \\
\lceil K \rfloor_L &= [\phi]^T \, [K]_L \, [\phi] = \lceil \omega^2 \rfloor \lceil M \rfloor \\
\lceil C \rfloor &= [\phi]^T \, [C] \, [\phi] = 2\lceil \zeta\omega \rfloor \lceil M \rfloor \\
\{P\} &= [\phi]^T \, \{p\}
\end{aligned} \tag{12}$$

The jth row of equation (11) is

$$\ddot{q}_j + 2\zeta_j \, \omega_j \, \dot{q}_j + \omega_j^2 \, q_j = \frac{P_j}{M_j} \tag{13}$$

The mean-square normal coordinate is simply

$$\overline{q_j^2} \simeq \frac{\pi S_p\ (\omega_j)}{4\ M_j^2\ \zeta_j\ \omega_j^3} = \frac{\pi\{\phi\}_j^T\ [S_p(\omega_j)]\ \{\phi\}_J}{4\ M_j^2\ \zeta_j\ \omega_j^3} \tag{14}$$

where $[S_p]$ is the cross-spectral density matrix of the excitation $\{p(t)\}$. The covariance matrix of the linear, generalized displacements is

$$[\overline{W_i W_j}]_L = \sum_k \{\phi\}_k \frac{\pi\{\phi\}_k^T\ [S_p(\omega_k)]\ \{\phi\}_k}{4\ M_K^2\ \zeta_k\ \omega_k^3} \{\phi\}_k^T \tag{15}$$

The diagonal terms $[\overline{W_i W_j}]_L$ are the mean-square, linear, generalized displacement $\overline{W_j^2}$. This initial estimate of $\overline{W_j^2}$ can now be used to compute the equivalent linear stiffness matrix $[K]_{EL}$ through equation (8). Then equation (7) is again transformed to the normal coordinates and has the form as

$$\lceil M \rfloor \{\ddot{q}\} + \lceil C \rfloor \{\dot{q}\} + \lceil K \rfloor \{q\} = \{P(t)\} \tag{16}$$

where $$\lceil K \rfloor = [\phi]^T \left([K]_L + [K]_{EL}\right) [\phi] = \lceil \Omega^2 \rfloor \lceil M \rfloor \tag{17}$$

The jth row of equation (17) is

$$\ddot{q} + 2\zeta_j\ \omega_j\ \dot{q}_j + \Omega^2\ q = \frac{P_j}{M_j} \tag{18}$$

and the displacement covariance matrix is given by

$$[\overline{W_i W_j}] = \sum_k \{\phi\}_k \frac{\pi\{\phi\}_k^T\ [S_p(\Omega_k)]\ \{\phi\}_k}{4\ M_k^2\ \zeta_k\ \Omega_K^2\ \omega_k} \{\phi\}_k^T \tag{19}$$

Convergence is considered achieved whenever the difference of the RMS generalized displacements satisfies the requirement

$$\left| \frac{(\mathrm{RMS}\ W_j)_{iter} - (\mathrm{RMS}\ W_j)_{iter-1}}{(\mathrm{RMS}\ W_j)_{iter}} \right| < 10^{-3}, \text{ for all } j \tag{20}$$

Once the RMS displacements are determined, the RMS deflection of the panel and the maximum RMS strain can be determined from equation (3) and the strain-displacement relations, respectively.

DEVELOPMENT OF GENERALIZED MATRICES AND COMPUTER PROGRAMS

The deflection of the panel is represented by

$$w(x,y,t) = h \sum_m \sum_n W_{mn}(t) \left\{ \left[\cos \frac{(m - 1)\pi x}{a} - \cos \frac{(m + 1)\pi x}{a} \right] \cdot \left[\cos \frac{(n - 1)\pi y}{b} - \cos \frac{(n + 1)\pi y}{b} \right] \right\} \tag{21}$$

The expression w satisfies the boundary condition for clamped edges:

$$\begin{aligned} w = w,_x &= 0 \quad \text{on } x = 0 \text{ and } a \\ w = w,_y &= 0 \quad \text{on } y = 0 \text{ and } b \end{aligned} \tag{22}$$

The stress function can be expressed in terms of the generalized displacement W_{mn} as

$$F = \overline{N}_x \frac{y^2}{2} + \overline{N}_y \frac{x^2}{2} + Eh^2 \sum_i \sum_j F_{ij} \cos \frac{i\pi x}{a} \cos \frac{j\pi y}{b} \tag{23}$$

where the coefficient F_{ij} is given by the expression

$$F_{ij} = \frac{1}{\left(\frac{i^2}{r} + j^2 r\right)^2} \sum_m \sum_n \sum_k \sum_l B_{ijmnkl} W_{mn} W_{kl} \tag{24}$$

in which B_{ijmnkl} are integers and $r = a/b$. The coefficients $\overline{N}_x$ and $\overline{N}_y$, and the integers B_{ijmnkl} are given explicitly in reference 7. The particular generalized displacements that are chosen to be nonzero in the convergence studies are shown in table 1.

Table 1. Generalized displacements for convergence studies.

Generalized Displacements	Number of terms 1	4	6	10	15
W_{11}	X	X	X	X	X
W_{13}		X	X	X	X
W_{31}		X	X	X	X
W_{33}		X	X	X	X
W_{15}			X	X	X
W_{51}			X	X	X
W_{35}				X	X
W_{53}				X	X
W_{17}				X	X
W_{71}				X	X
W_{55}					X
W_{37}					X
W_{73}					X
W_{19}					X
W_{91}					X

Utilizing the expressions for w and F, equations (21) and (23), respectively, and performing the integration of equation (5), the integral associated with the inertial force term in equation (1) has been derived as

$$\int_0^b \int_0^a \rho h w_{,tt} \, f_r \, g_s \, dxdy = \frac{\rho h^2 ab}{4} \Big(\ddot{W}_{r-2,s-2} - 2\,\ddot{W}_{r-2,s} - 2\,\ddot{W}_{r,s-2} + 4\,\ddot{W}_{r,s} - 2\,\ddot{W}_{r,s+2} - 2\,\ddot{W}_{r+2,s} + \ddot{W}_{r-2,s+2} + \ddot{W}_{r+2,s-2} + \ddot{W}_{r+2,s+2} \Big) \tag{25}$$

The generalized mass matrix $[M]$ in equation (6) using 15 terms in the deflection function is given by:

$$[M] = \frac{\rho h^2 ab}{4} \begin{array}{c} \begin{array}{ccccccccccccccc} W_{11} & W_{13} & W_{31} & W_{33} & W_{15} & W_{51} & W_{35} & W_{53} & W_{17} & W_{71} & W_{55} & W_{37} & W_{73} & W_{19} & W_{91} \end{array} \\ \begin{bmatrix} 4 & & & & & & & & & & & & & & \\ -2 & 4 & & & & & & & & & & & & & \\ -2 & 1 & 4 & & & & \text{symmetric} & & & & & & & & \\ 1 & -2 & -2 & 4 & & & & & & & & & & & \\ 0 & -2 & 0 & 1 & 4 & & & & & & & & & & \\ 0 & 0 & -2 & 1 & 0 & 4 & & & & & & & & & \\ 0 & 1 & 0 & -2 & -2 & 0 & 4 & & & & & & & & \\ 0 & 0 & 1 & -2 & 0 & -2 & 1 & 4 & & & & & & & \\ 0 & 0 & 0 & 0 & -2 & 0 & 1 & 0 & 4 & & & & & & \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 1 & 0 & 4 & & & & & \\ 0 & 0 & 0 & 1 & 0 & 0 & -2 & -2 & 0 & 0 & 4 & & & & \\ 0 & 0 & 0 & 0 & 1 & 0 & -2 & 0 & -2 & 0 & 1 & 4 & & & \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 & 0 & -2 & 1 & 0 & 4 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 1 & 0 & 4 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 1 & 0 & 4 \end{bmatrix} \end{array} \tag{26}$$

A subroutine program MASS, which generates the mass matrix, has been coded and verified. A listing of the MASS subroutine is given in the Appendix.

Similarly, the integrals associated with the linear stiffness terms in equation (1) yield

$$\int_o^b \int_o^a D \frac{\partial^4 w}{\partial x^4} f_r \, g_s \, dxdy = \frac{Dh\pi^4 ab}{4a^4}$$

$$\cdot \{ [(r-1)^4 + (r+1)^4] \left[(C_1 + 1)W_{r,s} - W_{r,s-2} - W_{r,s+2} \right]$$

$$+ (r-1)^4 \; [W_{r-2,s-2} + W_{r-2,s+2} - (C_1 + 1)W_{r-2,s}]\}$$

$$+ (r+1)^4 \; [W_{r+2,s+2} + W_{r+2,s-2} - (C_1 + 1) \; W_{r+2,s}]\} \tag{27}$$

$$\int_o^b \int_o^a D \frac{\partial^4 w}{\partial y^4} f_r \, g_s \, dxdy = \frac{Dh\pi^4 ab}{4b^4}$$

$$\cdot \{ [(s-1)^4 + (s+1)^4] \left[(C2 + 1)W_{r,s} - W_{r-2,s} - W_{r+2,s} \right]$$

$$+ (s-1)^4 \; [W_{r-2,s-2} + W_{r+2,s-2} - (C2 + 1)W_{r,s-2}]\}$$

$$+ (s+1)^4 \; [W_{r+2,s+2} + W_{r-2,s+2} - (C2 + 1) \; W_{r,s+2}]\} \tag{28}$$

$$\int_o^b \int_o^a 2D \frac{\partial^4 w}{\partial x^2 \, \partial y^2} f_r \, g_s \, dxdy = \frac{Dh\pi^4 ab}{a^2 b^2}$$

$$\cdot \{ (r-1)^2 \; (s+1)^2 \left[W_{r,s} - W_{r,s+2} - W_{r+2,s} + W_{r+2,s+2} \right]$$

$$+ (r-1)^2 \; (s+1)^2 \; [W_{r,s} - W_{r,s+2} - W_{r-2,s} + W_{r-2,s+2}$$

$$+ (r+1)^2 \; (s-1)^2 \left[W_{r,s} - W_{r,s-2} - W_{r+2,s} + W_{r+2,s-2} \right]$$

$$+ (r-1)^2 \; (s-1)^2 \left[W_{r,s} - W_{r,s-2} - W_{r-2,s} + W_{r-2,s-2} \right]\} \tag{29}$$

The generalized linear stiffness matrix $[K]_L$ in equation (6) using 15 terms in the deflection function has been derived. It can be expressed as the sum of the three submatrices as

$$[K]_L = \frac{Dh\pi^4 ab}{4}\left(\frac{1}{a^4}[K]_1 + \frac{1}{b^4}[K]_2 + \frac{4}{a^2b^2}[K]_3\right) \tag{30}$$

The nonzero elements of the three linear stiffness submatrices are given. Since the stiffness matrix is also symmetric, only the lower left-hand side elements are given. They are

$$[K]_1 = \begin{array}{c} \begin{array}{cccccccccc} W_{11} & W_{13} & W_{31} & W_{33} & W_{15} & W_{51} & W_{35} & W_{53} & W_{17} & W_{71} \end{array} \\ \left[\begin{array}{cccccccccc}
2^4(C_1+1) & & & & & & & & & \\
-2^4 & 2^4(C_1+1) & & & & & & & & \\
-2^4(C_1+1) & 2^4 & (2^4+4^4)(C_1+1) & & & & & & & \\
2^4 & -2^4(C_1+1) & -(2^4+4^4) & (2^4+4^4)(C_1+1) & & & & & & \\
0 & -2^4 & 0 & 2^4 & 2^4(C_1+1) & & & & & \\
0 & 0 & -4^4(C_1+1) & 4^4 & 0 & (4^4+6^4)(C_1+1) & & & & \\
0 & 2^4 & 0 & -(2^4+4^4) & -2^4(C_1+1) & 0 & (2^4+4^4)(C_1+1) & & \text{symmetric} & \\
0 & 0 & 4^4 & -4^4(C_1+1) & 0 & -(4^4+6^4) & 4^4 & (4^4+6^4)(C_1+1) & & \\
0 & 0 & 0 & 0 & -2^4 & 0 & 2^4 & 0 & 2^4(C_1+1) & \\
0 & 0 & 0 & 0 & 0 & -6^4(C_1+1) & 0 & 6^4 & 0 & (6^4+8^4)(C_1+1)
\end{array}\right] \end{array} \tag{31}$$

$$
[K]_2 = \begin{array}{c}
\begin{array}{cccccccccc}
W_{11} & W_{13} & W_{31} & W_{33} & W_{15} & W_{51} & W_{35} & W_{53} & W_{17} & W_{71}
\end{array} \\
\left[\begin{array}{cccccccccc}
2^4(C_2+1) & & & & & & & & & \\
-2^4(C_2+1) & (2^4+4^4)(C_2+1) & & & & & & & & \\
-2^4 & 2^4 & 2^4(C_2+1) & & & & & & & \\
2^4 & -(2^4+4^4) & -2^4(C_1+1) & (2^4+4^4)(C_2+1) & & & & & & \\
0 & -4^4(C_2+1) & 0 & 4^4 & (4^4+6^4)(C_2+1) & & & & & \\
0 & 0 & -2^4 & 2^4 & 0 & 2^4(C_2+1) & & & & \\
0 & 4^4 & 0 & -4^4(C_2+1) & -(4^4+6^4) & 0 & (4^4+6^4)(C_2+1) & & \text{symmetric} & \\
0 & 0 & 2^4 & -(2^4+4^4) & 0 & -2^4(C_2+1) & 4^4 & (2^4+4^4)(C_2+1) & & \\
0 & 0 & 0 & 0 & -6^4(C_2+1) & 0 & 6^4 & 0 & (6^4+8^4)(C_1+1) & \\
0 & 0 & 0 & 0 & 0 & -2^4 & 0 & 2^4 & 0 & 2^4(C_1+1)
\end{array}\right]
\end{array}
\tag{32}
$$

$$
[K]_3 = \begin{array}{c}
\begin{array}{cccccccccc} W_{11} & W_{13} & W_{31} & W_{33} & W_{15} & W_{51} & W_{35} & W_{53} & W_{17} & W_{71} \end{array} \\
\left[\begin{array}{cccccccccc}
(2\cdot 2)^2 & & & & & & & & & \\
-(2\cdot 2)^2 & (2\cdot 4)^2+(2\cdot 2)^2 & & \text{symmetric} & & & & & & \\
-(2\cdot 2)^2 & (2\cdot 2)^2 & (4\cdot 2)^2+(2\cdot 2)^2 & (4\cdot 4)^2 & & & & & & \\
(2\cdot 2)^2 & -(2\cdot 4)^2-(2\cdot 2)^2 & -(4\cdot 2)^2-(2\cdot 2)^2 & +2(4\cdot 2)^2+(2\cdot 2)^2 & & & & & & \\
0 & -(2\cdot 4)^2 & 0 & (2\cdot 4)^2 & (2\cdot 6)^2+(2\cdot 4)^2 & & & & & \\
0 & 0 & -(4\cdot 2)^2 & (4\cdot 2)^2 & 0 & (6\cdot 2)^2+(4\cdot 2)^2 & & & & \\
0 & (2\cdot 4)^2 & 0 & -(4\cdot 4)^2-(2\cdot 4)^2 & -(2\cdot 6)^2-(2\cdot 4)^2 & 0 & (4\cdot 6)^2+(2\cdot 6)^2+(4\cdot 4)^2+(2\cdot 4)^2 & & & \\
0 & 0 & (4\cdot 2)^2 & -(4\cdot 4)^2-(4\cdot 2)^2 & 0 & -(6\cdot 2)^2-(4\cdot 2)^2 & (4\cdot 4)^2 & (6\cdot 4)^2+(4\cdot 4)^2+(6\cdot 2)^2+(4\cdot 2)^2 & & \\
0 & 0 & 0 & 0 & -(2\cdot 6)^2 & 0 & (2\cdot 6)^2 & 0 & (2\cdot 8)^2+(2\cdot 6)^2 & \\
0 & 0 & 0 & 0 & 0 & -(6\cdot 2)^2 & 0 & (6\cdot 2)^2 & 0 & (8\cdot 2)^2+(6\cdot 2)^2
\end{array}\right]
\end{array}
\qquad (33)
$$

The nonzero elements of the linear stiffness matrix, $k_L(i,j)$ for $i,j \geq 11$, are

$$
\begin{aligned}
k_1(11,4) &= k_1(12,11) = 4^4 \\
k_1(12,5) &= k_1(14,12) = 2^4 \\
k_1(13,6) &= k_1(13,11) = 6^4 \\
k_1(11,7) &= -4^4(C_1 + 1) \\
k_1(12,7) &= -(2^4 + 4^4) \\
k_1(11,8) &= -(4^4 + 6^4)
\end{aligned}
\qquad (34) \text{ (cont'd)}
$$

$$k_1(13,8) = -6^4 \ (C_1 + 1)$$
$$k_1(12,9) = -2^4 \ (C_1 + 1)$$
$$k_1(14,9) = -2^4$$
$$k_1(13,10) = -(6^4 + 8^4)$$
$$k_1(15,10) = -8^4 \ (C_1 + 1)$$
$$k_1(11,11) = (4^4 + 6^4)(C_1 + 1)$$
$$k_1(12,12) = (2^4 + 4^4)(C_1 + 1)$$
$$k_1(13,13) = (6^4 + 8^4)(C_1 + 1)$$
$$k_1(15,13) = 8^4$$
$$k_1(14,14) = 2^4(C_1 + 1)$$
$$k_1(15,15) = (8^4 + 10^4)(C_1 + 1)$$
$$k_2(11,4) = k_2(13,11) = 4^4$$
$$k_2(12,5) = k_2(12,11) = 6^4$$
$$k_2(13,6) = k_2(15,13) = 2^4$$
$$k_2(11,7) = -(4^4 + 6^4)$$
$$k_2(12,7) = -6^4(C_2 + 1)$$
$$k_2(11,8) = -4^4(C_2 + 1)$$
$$k_2(13,8) = -(2^4 + 4^4)$$
$$k_2(12,9) = -(6^4 + 8^4)$$
$$k_2(14,9) = -8^4(C_2 + 1)$$
$$k_2(13,10) = -2^4(C_2 + 1)$$
$$k_2(15,10) = -2^4$$
$$k_2(11,11) = (4 + 6)(C_2 + 1)$$
$$k_2(12,12) = (6^4 + 8^4)(C_2 + 1)$$
$$k_2(14,12) = 8^4$$
$$k_2(13,13) = (2^4 + 4^4)(C_2 + 1)$$
$$k_2(14,14) = (8^4 + 10^4)(C_2 + 1)$$
$$k_2(15,15) = 2^4(C_2 + 1)$$
$$k_3(11,4) = (4 \cdot 4)^2$$
$$k_3(12,5) = k_3(13,6) = (2 \cdot 6)^2$$
$$k_3(11,7) = k_3(11,8) = -(4 \cdot 4)^2 - (4 \cdot 6)^2$$
$$k_3(12,7) = k_3(13,8) = -(2 \cdot 6)^2 - (4 \cdot 6)^2$$
$$k_3(12,9) = k_3(13,10) = -(2 \cdot 6)^2 - (2 \cdot 8)^2$$
$$k_3(14,9) = k_3(15,10) = -(2 \cdot 8)^2$$
$$k_3(11,11) = (6 \cdot 6)^2 + 2(6 \cdot 4)^2 + (4 \cdot 4)^2 \tag{34}$$
$$k_3(12,11) = k_3(13,11) = (4 \cdot 6)^2$$

(cont'd)

$$k_3(12,12) = k_3(13,13) = (4\cdot8)^2 + (2\cdot8)^2 + (4\cdot6)^2 + (2\cdot6)^2 \qquad (34)$$
$$k_3(14,14) = k_3(15,15) = (2\cdot8)^2 + (2\cdot10)^2 \qquad \text{(concl'd)}$$

where

$$C_1 = \begin{cases} 2 & \text{for } s = 1 \\ 1 & \text{for } s \neq 1 \end{cases}$$

$$C_2 = \begin{cases} 2 & \text{for } r = 1 \\ 1 & \text{for } r \neq 1 \end{cases} \qquad (35)$$

A subroutine program LSTF which generates the linear stiffness matrix has been coded and verified. A listing of the LSTF subroutine is presented in the Appendix.

Derivation of the generalized equivalent linear stiffness matrix $[K]_{EL}$ in equation (7a) has been initiated. It is in good progress. Continuing research effort will be devoted to the following tasks:

(1) Completion of the derivation of equivalent linear stiffness;

(2) Application of eigen solution and coordinate transformation;

(3) Determination of mean-square linear generalized displacement;

(4) Implementation of the iterative process;

(5) Derivation of strains computation;

(6) Coding, debugging, and verifying the complete computer program;

(7) Convergence studies; and

(8) Generation of design charts.

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APPENDIX

LISTINGS OF THE MASS AND LSTF SUBROUTINES

```
      SUBROUTINE MASS(AL,BL,H,RHO,SM,NTERM)
      DIMENSION SM(NTERM,NTERM)
C
C THIS SUBROUTINE GENERATES THE SYSTEM MASS MATRIX OF THE PANEL USING
C (NTERM) TERMS IN THE DEFLECTION FUNCTION
C AL=PANEL LENGTH.
C BL=PANEL WIDTH
C H=PANEL THICKNESS
C RHO=MASS DENSITY
C SM(NTERM,NTERM)=SYSTEM OR GENERALIZED MASS MATRIX
C NTERM=1, 4, 6, 10, OR 15
C
      COEF=0.25*RHO*H*H*AL*BL
C INITIALIZED THE MASS MATRIX
      DO 10 I=1,NTERM
      DO 10 J=1,NTERM
      SM(I,J)=0.0
   10 CONTINUE
      SM(1,1)=4.0*COEF
      IF (NTERM .EQ. 1) GO TO 20
C
      SM(1,2)=-2.0*COEF
      SM(1,3)=-2.0*COEF
      SM(1,4)=COEF
      SM(2,2)=4.0*COEF
      SM(2,3)=COEF
      SM(2,4)=-2.0*COEF
      SM(3,3)=4.0*COEF
      SM(3,4)=-2.0*COEF
      SM(4,4)=4.0*COEF
      IF (NTERM .EQ. 4) GO TO 20
C
      SM(2,5)=-2.0*COEF
      SM(3,6)=-2.0*COEF
      SM(4,5)=COEF
      SM(4,6)=COEF
      SM(5,5)=4.0*COEF
      SM(6,6)=4.0*COEF
      IF (NTERM .EQ. 6) GO TO 20
C
      SM(2,7)=COEF
      SM(3,8)=COEF
      SM(4,7)=-2.0*COEF
      SM(4,8)=-2.0*COEF
      SM(5,7)=-2.0*COEF
      SM(5,9)=-2.0*COEF
      SM(6,8)=-2.0*COEF
      SM(6,10)=-2.0*COEF
      SM(7,7)=4.0*COEF
      SM(7,8)=COEF
      SM(7,9)=COEF
      SM(8,8)=4.0*COEF
      SM(8,10)=COEF
      SM(9,9)=4.0*COEF
      SM(10,10)=4.0*COEF
      IF (NTERM .EQ. 10) GO TO 20
```

```
      C
               SM(4,11)=COEF
               SM(5,12)=COEF
               SM(6,13)=COEF
               SM(7,11)=-2.0*COEF
               SM(7,12)=-2.0*COEF
               SM(8,11)=-2.0*COEF
               SM(8,13)=-2.0*COEF
               SM(9,12)=-2.0*COEF
               SM(9,14)=-2.0*COEF
               SM(10,13)=-2.0*COEF
               SM(10,15)=-2.0*COEF
               SM(11,11)=4.0*COEF
               SM(11,12)=COEF
               SM(11,13)=COEF
               SM(12,12)=4.0*COEF
               SM(12,14)=COEF
               SM(13,13)=4.0*COEF
               SM(13,15)=COEF
               SM(14,14)=4.0*COEF
               SM(15,15)=4.0*COEF
          20   CONTINUE
               DO 30 J=1,NTERM
               DO 30 I=J,NTERM
               SM(I,J)=SM(J,I)
          30   CONTINUE
               RETURN
               END
```

```
SUBPROGRAMS CALLED


SCALARS AND ARRAYS [ "*" NO EXPLICIT DEFINITION - "%" NOT REFERENCED ]

*NTERM  1      *COEF   2      *H      3      *BL     4      SM      5
*J      6      .S0003  7      .S0002  10     .S0001  11     .S0000  12
*RHO    13     *AL     14     .I0002  15     .I0001  16     *I      17
 .I0000 20

TEMPORARIES

 .A0016 21

MASS      [ NO ERRORS DETECTED ]
```

```
      SUBROUTINE LSTF(AL,BL,H,D,PI,SK,NTERM)
      DIMENSION SK(NTERM,NTERM)
C
C  THIS SUBROUTINE GENERATES THE SYSTEM LINEAR STIFFNESS MATRIX
C  USING (NTERM) TERMS IN THE DEFLECTION FUNCTION FOR THE PANEL
C  AL=PANEL LENGTH
C  BL=PANEL WIDTH
C  H=PANEL THICKNESS
C  D=E*H**3/((12.0*(1-V**2))=BENDING RIGIDITY
C  V=POISSON'S RATIO
C  SK(NTERM,NTERM)=SYSTEM OR GENERALIZED STIFFNESS MATRIX
C  NTERM=1, 4, 6, 10, OR 15
C
      PI4=PI*PI*PI*PI
      COEF=4.0*D*H*PI4*AL*BL
      A4=1.0/(AL*AL*AL*AL)
      B4=1.0/(BL*BL*BL*BL)
      AB2=4.0/(AL*AL*BL*BL)
C  C1=C1 FOR S<>1,C1=CC1 FOR S=1
C  C2=C2 FOR R<>1,C2=CC2 FOR R=1
      C1=1.0
      C2=1.0
      CC1=2.0
      CC2=2.0
C  INITIALIZED THE STIFFNESS MATRIX
      DO 10 I=1,NTERM
      DO 10 J=1,NTERM
      SK(I,J)=0.0
   10 CONTINUE
      SK(1,1)=((CC1+1.0)*A4+(CC2+1.0)*B4+AB2)*COEF
      IF(NTERM .EQ. 1) GO TO 20
C
      SK(1,2)=-((CC2+1.0)*B4+A4+AB2)*COEF
      SK(1,3)=-((CC1+1.0)*A4+B4+AB2)*COEF
      SK(1,4)=(A4+B4+AB2)*COEF
      SK(2,2)=(17.0*(CC2+1.0)*B4+(C1+1.0)*A4+5.0*AB2)*COEF
      SK(2,3)=(A4+B4+AB2)*COEF
      SK(2,4)=-((C1+1.0)*A4+17.0*B4+5.0*AB2)*COEF
      SK(3,3)=(17.0*(CC1+1.0)*A4+(C2+1.0)*B4+5.0*AB2)*COEF
      SK(3,4)=-((C2+1.0)*B4+17.0*A4+5.0*AB2)*COEF
      SK(4,4)=(17.0*(C1+1.0)*A4+17.0*(C2+1.0)*B4+25.0*AB2)*COEF
      IF(NTERM .EQ. 4) GO TO 20
C
      SK(2,5)=-(16.0*(CC2+1.0)*B4+A4+4.0*AB2)*COEF
      SK(3,6)=-(16.0*(CC1+1.0)*A4+B4+4.0*AB2)*COEF
      SK(4,5)=(A4+16.0*B4+4.0*AB2)*COEF
      SK(4,6)=(16.0*A4+B4+4.0*AB2)*COEF
      SK(5,5)=((C1+1.0)*A4+97.0*(CC2+1.0)*B4+13.0*AB2)*COEF
      SK(6,6)=(97.0*(CC1+1.0)*A4+(C2+1.0)*B4+13.0*AB2)*COEF
      IF(NTERM .EQ. 6) GO TO 20
C
      SK(2,7)=(A4+16.0*B4+4.0*AB2)*COEF
      SK(3,8)=(16.0*A4+B4+4.0*AB2)*COEF
      SK(4,7)=-(17.0*A4+16.0*(C2+1.0)*B4+20.0*AB2)*COEF
      SK(4,8)=-(16.0*(C1+1.0)*A4+17.0*B4+20.0*AB2)*COEF
      SK(5,7)=-((C1+1.0)*A4+97.0*B4+13.0*AB2)*COEF
```

```
      SK(5,9)=-(A4+81.0*(CC2+1.0)*B4+9.0*AB2)*COEF
      SK(6,8)=-(97.0*A4+(C2+1.0)*B4+13.0*AB2)*COEF
      SK(6,10)=-(81.0*(CC1+1.0)*A4+B4+9.0*AB2)*COEF
      SK(7,7)=(17.0*(C1+1.0)*A4+97.0*(C2+1.0)*B4+65.0*AB2)*COEF
      SK(7,8)=16.0*(A4+B4+AB2)*COEF
      SK(7,9)=(A4+81.0*B4+9.0*AB2)*COEF
      SK(8,8)=(97.0*(C1+1.0)*A4+17.0*(C2+1.0)*B4+65.0*AB2)*COEF
      SK(8,10)=(81.0*A4+B4+9.0*AB2)*COEF
      SK(9,9)=((C1+1.0)*A4+337.0*(CC2+1.0)*B4+25.0*AB2)*COEF
      SK(10,10)=(337.0*(CC1+1.0)*A4+(C2+1.0)*B4+25.0*AB2)*COEF
      IF(NTERM .EQ. 10) GO TO 20
C
      SK(4,11)=16.0*(A4+B4+AB2)*COEF
      SK(5,12)=(A4+81.0*B4+9.0*AB2)*COEF
      SK(6,13)=(B4+81.0*A4+9.0*AB2)*COEF
      SK(7,11)=-(16.0*(C1+1.0)*A4+97.0*B4+52.0*AB2)*COEF
      SK(7,12)=-(17.0*A4+81.0*(C2+1.0)*B4+45.0*AB2)*COEF
      SK(8,11)=-(97.0*A4+16.0*(C2+1.0)*B4+52.0*AB2)*COEF
      SK(8,13)=-(81.0*(C1+1.0)*A4+17.0*B4+45.0*AB2)*COEF
      SK(9,12)=-((C1+1.0)*A4+337.0*B4+25.0*AB2)*COEF
      SK(9,14)=-(A4+256.0*(CC2+1.0)*B4+16.0*AB2)*COEF
      SK(10,13)=-(337.0*A4+(C2+1.0)*B4+25.0*AB2)*COEF
      SK(10,15)=-(256.0*(CC1+1.0)*A4+B4+16.0*AB2)*COEF
      SK(11,11)=(97.0*(C1+1.0)*A4+97.0*(C2+1.0)*B4+169.0*AB2)*COEF
      SK(11,12)=(16.0*A4+81.0*B4+36.0*AB2)*COEF
      SK(11,13)=(81.0*A4+16.0*B4+36.0*AB2)*COEF
      SK(12,12)=(17.0*(C1+1.0)*A4+337.0*(C2+1.0)*B4+125.0*AB2)*COEF
      SK(12,14)=(A4+256.0*B4+16.0*AB2)*COEF
      SK(13,13)=(337.0*(C1+1.0)*A4+17.0*(C2+1.0)*B4+125.0*AB2)*COEF
      SK(13,15)=(B4+256.0*A4+16.0*AB2)*COEF
      SK(14,14)=((C1+1.0)*A4+881.0*(CC2+1.0)*B4+41.0*AB2)*COEF
      SK(15,15)=((C2+1.0)*B4+881.0*(CC1+1.0)*A4+41.0*AB2)*COEF
   20 CONTINUE
      DO 30 J=1,NTERM
      DO 30 I=J,NTERM
      SK(I,J)=SK(J,I)
   30 CONTINUE
      RETURN
      END
```

SUBPROGRAMS CALLED

SCALARS AND ARRAYS ["*" NO EXPLICIT DEFINITION - "%" NOT REFERENCED]

*A4	1	*NTERM	2	*COEF	3	*PI	4	*H	5
*CC1	6	*BL	7	*J	10	*D	11	*AB2	12
.S0003	13	.S0002	14	.S0001	15	.S0000	16	*B4	17
*CC2	20	*C2	21	*PI4	22	*AL	23	SK	24
.I0002	25	.I0001	26	*I	27	.I0000	30	*C1	31